A Review of Load Flow Methodologies for Constrained Networks: A South African Case Study

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Abstract
South Africa, along with the rest of the world, is becoming increasingly dependent on variable renewable energy sources which increase the uncertainties associated with operating and planning a power system. Traditionally, deterministic load flow analysis was used for modelling power systems, but it has been shown to be unable to represent the uncertainties of network states well. Commonly, probabilistic load flow analysis is suggested as an alternative to DLF. This review paper provides a comparative analysis of probabilistic load flow analysis (PLF) and DLF at the hand of prevalent literature. The advantages and disadvantages of the respective methodologies are emphasised specifically in relation to modern power networks with a large share of renewable generators to determine which is better suited to constrained systems. This paper discusses certain commonly used PLF methodologies along with their typical applications and their suitability for capacity planning in South Africa. PLF is found to be preferable to DLF for decision-making purposes, as it provides users with probabilistic information representing risks associated with modelling outcomes. PLF, however, carries a significantly larger computational burden and requires more detailed data on various network components. The added information provided by PLF potentially allows for more economic system designs during power system planning as well as more efficient utilisation of existing infrastructure during operations, as it provides a measure of the risks associated with making these decisions. Of the PLF methodologies considered, numerical PLF methods show the greatest maturity for use in system operation and network planning purposes. Analytical and approximate methods have been shown useful for specific scenarios but seem limited in their aptness for the South African system due to the amount of input variables and the complexity of correlations that can be considered, respectively.

Keywords: probabilistic load flow, review article, variable renewable energy.

1. Introduction
South Africa has experienced a significant increase in the installed capacity of variable renewable energy sources (VRES), similar to the global trend of the electricity sector (IEA, 2021). To comply with carbon emission commitments and to alleviate the country's ongoing energy crisis, further expansion of VRES generation in the country is urgently required (Mantashe, 2019). The transmission network is however underdeveloped at locations with the best resource potential – particularly with the Northern Cape, indicated in Figure 1 as NC – due to the low population density and relatively great distances to major load centres. As such, the transmission network in these areas has been designated as being unable to host further generation capacity (Matshidza et al., 2022). While there are projects to expand the connection capacity in these regions, the lead times for these prevent the areas from being available within the next few years (Scheppers, 2021). A possible solution to this congestion would be to reconsider how congestion is determined by changing the load flow methodologies used to calculate it.

Traditionally, power system hosting capacity was analysed using deterministic load flow (DLF) methodologies where input variables and results are represented by single values (Gross, 1979). To account for uncertainties, engineers design a set of worst-case scenarios and then plan around these (Kirschen & Jayaweera, 2007). It has however been noted widely in the literature that DLFs provide a highly conservative representation of the hosting capacity of transmission networks – particularly within the context of large penetrations of VRES (Kirschen & Jayaweera, 2007; Leite da Silva & de Castro, 2019)

A common suggestion for an alternative methodology is probabilistic load flows (PLFs) (Chen, Chen & Bak-Jensen, 2008; Prusty & Jena, 2017; Dalton, Bekker & Koivisto, 2021) which was proposed by Borkowska in the 1970s (Borkowska, 1974). PLF assigns probability density functions (PDFs) to the input variables and also presents the output probabilistically (Borkowska, 1974) which provides a risk perspective on operational and planning tasks (Reppen et al., 1975; Chen, Chen & Bak-Jensen, 2008)

A third load flow technique, possibilistic load flow, was designed to compensate for situations where there is insufficient input data to perform a PLF (Zadeh, 1978; Aien, Rashidinejad & Fotuhi-Firuzabad, 2014). It shows promise for systems for severe uncertainty, but is as yet not mature enough yet for general use (Aien, Rashidinejad & Fotuhi-Firuzabad, 2014; Aien, Hajebrahimi & Fotuhi-Firuzabad, 2016) and is therefore not considered in this review.

Within the context of South Africa's constrained transmission network, this study firstly compares DLF and PLF methodologies at the hand of the current state of the literature, particularly in terms of their aptness for determining transmission network capacity in contemporary power systems – characterised by high levels of installed VRESs. Secondly, an overview and summary of the most prominent PLF techniques is provided, specifically with reference to their suitability and maturity for use in an operational environment.

The South African power system has a few characteristics to bear in mind while reviewing the load flow methodologies:

- The north-eastern parts of the country have a well-developed network with significant generation and load present, while the rest of the country is fed from this hub by long transmission corridors, leading to weaker portions. Figure 1 shows the transmission network overlaid on a provincial map of the country. Note the denseness of the network in the north when compared to the rest of the country (Scheppers, 2021).
- The south-western parts of South Africa have the best solar and wind resource potential, but a fairly poorly developed transmission network.

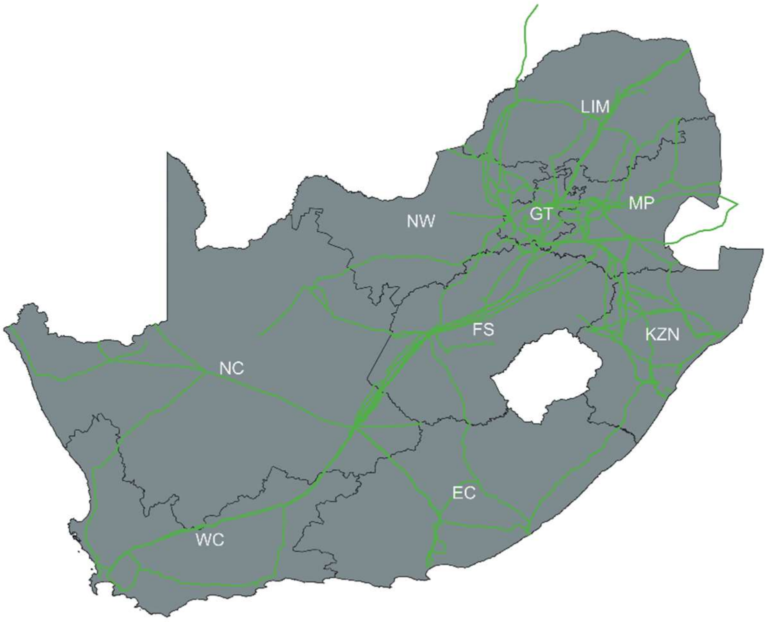

Figure 1. Map of South African transmission network overlaid on a provincial map

- VRES plants currently account for around 10% of the national installed generation capacity, with plans in place to increase this to 30% by 2030 (Mantashe, 2019).
- The electricity sector is vertically integrated, but is transitioning to a free-market system (Mantashe, 2019).

Though this study is primarily focused on the South African context, it should be noted that the problem of increasingly constrained transmission networks is by no means unique(Mararakanye & Bekker, 2019). The world is moving towards VRES and ensuring that the techniques used in analysing transmission network capacity are of utmost importance in planning of future energy systems and ensuring secure power supplies, as well as ensuring cost-optimal operations of power systems.

The rest of the paper reads as follows: section two compares DLF and PLF methods in the context of systems with significant amounts of VRESs, section three then investigates possible PLF methodologies to determine which would be most suited for the South African context and section four provides closing statements as well as a discussion on the research to follow.

2. Probabilistic load flow compared to deterministic load flow

The following section discusses the advantages and disadvantages of using PLF, compared to DLF, to analyse power systems.

2.1. Advantages of using probabilistic load flow

The main advantage of using PLF is that it provides more information to the engineers on which to base decisions. It does this by incorporating uncertainties into load flow calculations and quantifying the uncertainty of the output as well. There is no consistent classification system of power system uncertainties, (Aien, Hajebrahimi & Fotuhi-Firuzabad, 2016; Milanović, 2017; Prusty & Jena, 2017; Chihota & Gaunt, 2018a), as categorisations are more useful for specific conversations than others, so for this discussion power system uncertainties are grouped as:

- Input uncertainty : Deviances in actual versus predicted load flow in the system which can be due to forecast or measurement errors of generator and load power, unexpected breakdowns of generators and transmission network components, or incorrect load forecast models amongst other things.

- Network uncertainty: Deviances in the topology and the physical properties of the transmission and distribution networks from what is modelled.

As the penetration of VRES, or similar non-dispatchable technologies, increase in a power system, the input uncertainties of the system increases as well, making it difficult to sensibly choose system snapshots for DLF analyses, as determining the likelihood of a specific system state becomes more complex, possibly resulting to network planning decisions being based on scenarios with a very low probability of occurring. Basing planning decisions on unlikely scenarios may result in the network being over-built and underutilised, ultimately at significant cost to the consumer without appreciable increase in the security of power supplied to the consumer. Using PLF methods provide a measure of the risk associated with making decisions by specifying the likelihood of a system state to take place.

2.2. Disadvantages of using probabilistic load flow

As the main advantage of using PLF is increased information, the primary disadvantage is the increased computational load. The increased load is either due to the added complexity of performing a PLF or to the amount of extra simulations that have to be run. A significant amount of effort has been invested in lowering the computational time of PLFs while still ensuring sufficient accuracy of solutions (Hu & Wang, 2006; Fan et al., 2012; Chihota & Gaunt, 2018b; Dalton, Bekker & Koivisto, 2021) with mixed results. Generally, either the computation time is directly correlated to the system size, or consistent computation times are achieved at the cost of the range of input PDFs and the type of correlations between them that can be considered (Aien, Hajebrahimi & Fotuhi-Firuzabad, 2016).

Along with the increased computational demand, PLF has a higher data demand than DLF. Sufficient generation, load, and network time-series data is required to construe PDFs for these input parameters. Calculating the correlations between input parameters is crucial to ensuring accurate results (Widén, Shepero & Munkhammar, 2017; Dalton, Bekker & Koivisto, 2021), but requires that the time-series data must either be from specific sites, or that sites are aggregated according to similar output behaviour (Janse van Vuuren, Vermeulen & Bekker, 2019; Dalton, Bekker & Koivisto, 2021). Though it would be possible to model the output from VRES sites using statistical distributions, time-series data is still preferred as the statistical model for individual VRES sites can vary dramatically between sites (Liu et al., 2016), with research on improving the statistical models used for describing VRESs still ongoing (Janse van Vuuren, Vermeulen & Bekker, 2019; Farmer & Rix, 2020).

2.3. Challenges to the adoption of probabilistic load flow

Along with the disadvantages noted above, there are several challenges to the broad-scale adoption of PLF. These challenges are not caused by factors inherent to the PLF methodology, but are related external factors like the inertia of adopting new procedures in industrial settings. Such challenges include the following:

- Deregulation of power systems: There is a shift in power systems from integrated monopolies to open market models where the generation and distribution of power is managed by separate entities. To ensure that the distribution entities remain able to balance generation and demand, the generators are required to provide estimates of future power generation. Some system operators only require point estimates from the generators, i.e. a single expected power output value for each time unit, usually an hour, (NERSA, 2016), which makes it difficult for to perform PLFs for operational purposes, as converting deterministic values to PDFs significantly impacts the accuracy of the PLF solution and is currently limited in the types of distributions that this can be done with (Aien, Rashidinejad & Fotuhi-Firuzabad, 2014).
- Several commercial load flow software packages do not support PLF functionality, or only implements Monte Carlo methods (DIgSILENT GmbH, 2019; Eaton, 2020; PowerWorld Corporation, 2022). Monte Carlo simulations, though most rigorous,

are significantly slower than performing DLFs, as it effectively runs thousands of DLFs to determine an output.

- PLF techniques are rarely taught to power system engineers (Glover, 2008; Wang, Song & Irving, 2008; Mararakanye & Bekker, 2019). Engineers are therefore less comfortable, and less proficient, with this method and thus less likely to use it.

3. Summary of probabilistic load flow methods

PLF is commonly categorised into three types of methods, numerical, analytical and approximate. The following section briefly describes each method, along with the current state of that method. A visual summary of the different PLF and DLF methods are presented in Figure 2, which details the common methods for the two types of load flow. The DLF methods are included for completeness, but are not discussed in this paper.

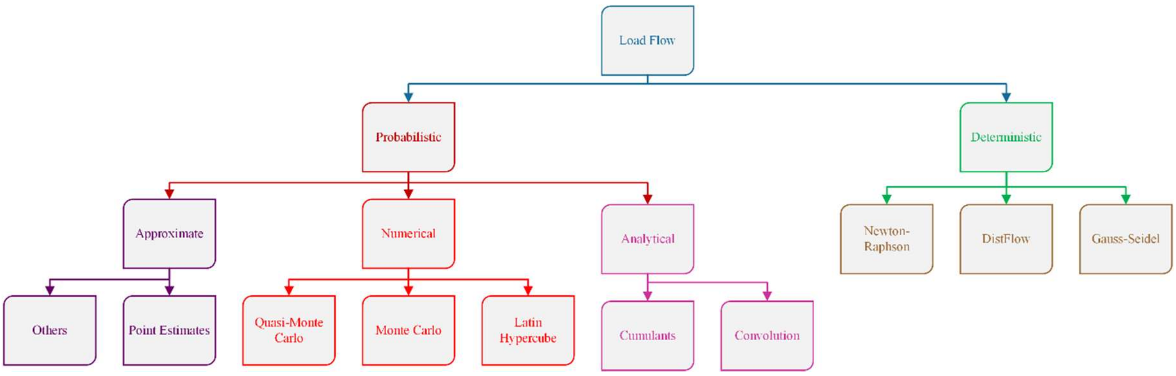

Figure 2. Breakdown of load flow methods.

3.1. Numerical methods

Numerical methods perform a large number of DLF simulations, with the values of the input deterministic states sampled from the PDFs of the input random variables (Ramadhani et al., 2020) and the result for each DLF then forms a data point for the PLF output. This method is both robust and accurate and is therefore commonly used as reference when considering the performance of new methods (Fan et al., 2013). The accuracy of numerical methods come at the cost of long computation times due to the amount of simulations that are required to build the output PDFs.

Numerical PLF sub-methods are primarily characterised by different sampling techniques.

3.1.1. Monte Carlo sampling

Proposed in 1975 by (Sufana, Heydt & Sauer, 1975) and (Dopazo, Klitin & Sasson, 1975) as an alternative to linearising the load flow equations, this method draws samples from the input PDFs using a simple Monte Carlo method, where it is equally probable for any sample to appear. A naïve approach runs a fixed amount of load flows but, as samples are equiprobable, there is no way of determining whether the output sufficiently covers the sample space using this technique. The preferred approach is thus to have a stopping criteria based on a coefficient of variation, the calculation of which is discussed in (Prusty & Jena, 2017). Note that using the coefficient of variation as stopping criteria, rather than the amount of simulations, does not necessarily reduce the computation time of the method, it only insures a certain level of accuracy.

3.1.2. Quasi-Monte Carlo sampling

In an attempt to reduce the computation time without reducing accuracy, (Singhee & Rutenbar, 2010) suggests adopting Sobol and Halton sampling sequences for use in integrated circuit analysis and (Huang et al., 2013) applies it to grid scale analyses. The idea behind these sampling sequences are that rather than taking equiprobable samples, equidistant samples are taking, thereby covering the sample space more evenly. These methods have been coined quasi-Monte Carlo simulations and have been shown to achieve the same level of output accuracy using fewer samples than standard Monte Carlo techniques, allowing them to be faster (Fang et al., 2014; Gu et al., 2014) at the cost of a slightly more complex initial setup.

3.1.3. Latin-hypercube sampling

Similarly to quasi-Monte Carlo techniques, Latin-hypercube sampling attempts to reduce the amount of simulations that are required for accurate results by choosing the input samples with greater care. Latin Hypercube sampling ensures efficient coverage of the sample space by first dividing the sample space into N, equal, non-overlapping intervals and then picking a single value from each of these intervals, either at random or by using the midpoint (Stein, 1987). Results are therefore achieved faster; as the sample size is effectively reduced, but it requires the cumulative density function of all the input variables and assumes that these variables follow a typical marginal distribution (Cai, Shi & Chen, 2014).

3.2. Analytical methods

The basis of the analytical techniques is finding and solving a set of mathematical expressions that relate the output PDF to the input PDFs. Due to the non-linear nature of load flow equations, along with the complex correlations between power flow inputs, (Allan & Al-Shakarchi, 1976; Anders, 1989) these methods generally require simplifying assumptions, linearisation of the load flow equations and linear correlation between inputs among others, to be feasible (Chen, Chen & Bak-Jensen, 2008). Analytical methods show favourable computational times when compared to numerical methods, but are less robust due to the simplifying assumptions (Ramadhani et al., 2020). Zhang and Lee use the cumulant PLF method as a screening tool for expansion planning, (Zhang & Lee, 2004), while (Anastasiadis, Voreadi & Hatziargyriou, 2011) uses it to determine the effects of different electric vehicle charging strategies on the Greek power network.

Two of the best known analytical techniques are the convolution and cumulant methods. Approximate methods can also be considered as an analytical technique but will be discussed separately in this paper, as this is common in literature reviews (Aien, Hajebrahimi & Fotuhi-Firuzabad, 2016; Chihota & Gaunt, 2018a; Hasan, Preece & Milanović, 2019; Ramadhani et al., 2020), in subsection 3.3. The rest of this subsection describes the convolution and cumulant methods in greater detail.

3.2.1. Convolution method

Proposed by Borkowska, in the original description of PLF (Borkowska, 1974), this method calculates the output PDF as the convolution of all the input PDFs. The computational time is therefore strongly dependent on the size of the system under consideration. Attempts have been made to reduce the computational burden for larger systems using different techniques (Allan, Da Silva & Burchett, 1981; Valverde, Saric & Terzija, 2012; Prusty & Jena, 2015), and though these do reduce computational time the computational load presented by these methods are still significant. The convolution method is no longer in common use.

3.2.2. Cumulant method

The cumulant method to solve load flow problems was introduced in 1986 by Sanabria and Dillon (Sanabria & Dillon, 1986). By converting the moments of the input variables to cumulants, it becomes possible to calculate the net power injected at a node as the sum of the cumulants at the node (Sanabria & Dillon, 1986), which in several studies has been shown to be faster and more accurate than convolving the input distributions (Prusty & Jena, 2015; Aien, Hajebrahimi & Fotuhi-Firuzabad, 2016; Hasan, Preece & Milanović, 2019). The output cumulants are converted back to a PDF using an expansion series (Fan et al., 2013). Different expansion techniques are compared in (Fan et al., 2012) whereby it was concluded that there is no simple, generic algorithm to determine which expansion series to use, but that it will require a case by case analysis.

3.3. Approximate methods

Approximate methods attempt to estimate the input PDFs instead of formulating them from time-series data (Prusty & Jena, 2017). It therefore does not require complete information about the shape of the PDF, but only central moments. Approximate methods then perform DLFs with these points, which produces the moments of the output distributions (Hasan, Preece & Milanović, 2019). Similar to analytical techniques, approximate methods require a processing step to turn the output moments into PDFs. Approximate methods are computationally efficient and easy to implement, but their performance decreases as the number of input variables and the complexity of the input PDFs increase (Ramadhani et al., 2020). Approximate methods are also limited in the input correlations that they can consider.

3.3.1. Point estimation methods

Point estimation methods require the calculation of the concentration of input samples, the sample point with its corresponding weight, which is then used to approximate key statistical moments of the output function (Ramadhani et al., 2020). There are various forms of point estimation, named after the amount of DLFs that each must run, which can consider differing amounts of input variables (Prusty & Jena, 2015). The forms are compared in (Caramia, Carpinelli & Varilone, 2010) and it is found that 2m+1 point estimation provides sufficient accuracy and ability to consider multiple input variables. Zhang et al. successfully applies a point estimation scheme that has been adapted to consider correlations between generation sources on a test network in (Zhang et al., 2014), though concludes that it is susceptible to the certainty of input PDFs.

3.3.2. Unscented transformation method

Similar to the point estimation technique, the unscented transformation deterministically creates a suitable amount of input samples such that the key statistical moments of the samples correspond to the input variable PDF (Aien, Hajebrahimi & Fotuhi-Firuzabad, 2016). The primary advantage of this above point estimation is that the unscented transformation considers correlations between input variables efficiently (Ramadhani et al., 2020).

3.4. Suitability of PLF methods for calculating hosting capacity

The literature suggests that it would be possible to use numerical, analytical and approximate methods for calculating hosting capacity in power systems, though with limitations in the range of applications that can be considered.

Using the characteristics of the South African power system as criteria, the suitability of the PLF methods are described below:

- Numerical methods: Literature suggests that these can be used for any type of power system, with the solution time being largely independent of the system size. It would therefore be suitable for the large South African system with its mixture of dispatchable and VRES generation. The long computational times could possibly be restrictive when using it for operational planning.
- Analytical methods: Convolution methods are unsuited for anything but back of the envelope calculations due to the linearising assumptions. Cumulant techniques show promise, but the output requires verification with numerical methods, and should therefore only be considered if numerical methods do not provide solutions of adequate accuracy for use in operational time frames.

- Approximate methods: The limited ability of these techniques to consider input correlations make them less suited for the South African context with its growing penetration of VRES generation.

4. Conclusion

With the increased use of VRES in power systems, the need for load flow methodologies that represent uncertainties associated with operational states are rising. This review paper firstly provides a comparison of DLF and PLF methodologies. It then gives an overview of certain commonly used PLF methodologies was provided along with a discussion on their possible applications.

PLF has the advantage of representing the stochastic nature of power systems. This enables engineers to better understand the real behaviour of the power system under consideration and therefore assists with planning and operation of power systems. PLF has the disadvantage of being relatively complex to implement and running PLF simulations are significantly more time-consuming and computationally intensive when compared to DLF simulations. Furthermore, there are a number of challenges in the context of current power systems which make it difficult to adopt PLF, such as it not being commonly taught to power system engineers or supported by many commercial software packages.

Literature suggests that numerical PLF methodologies are currently sufficiently matured whereby they can be applied to power systems for planning and operational use. Analytical techniques are useful for faster, back-of-the envelope type of analyses and could possibly be used during operational time frames. Approximate methods still require some expansion to make them practically applicable on large scale power systems, but are useable for smaller power systems.

From the literature, it seems evident that using PLF to analyse the hosting capacity of South African transmission network, will have a marked impact on the perceived capacity of the system. It will therefore be useful to do a study in which a PLF methodology is applied to the system. Given the maturity of numerical methods, it is suggested that this is attempted first.

This review is expected to be of use to power system engineers and managers, as it provides a high level overview of PLF as an alternative to DLF for analysis purposes.

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