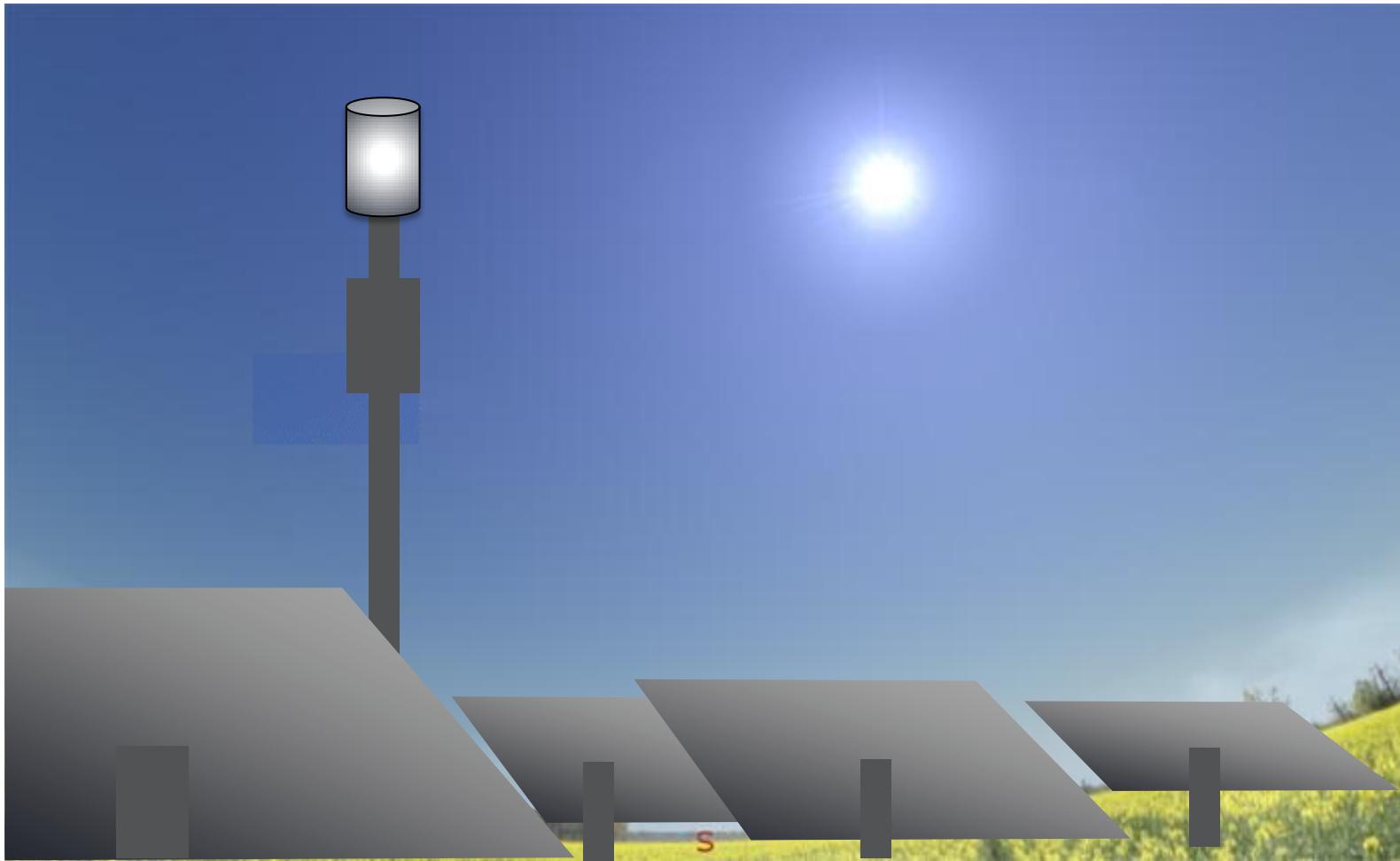


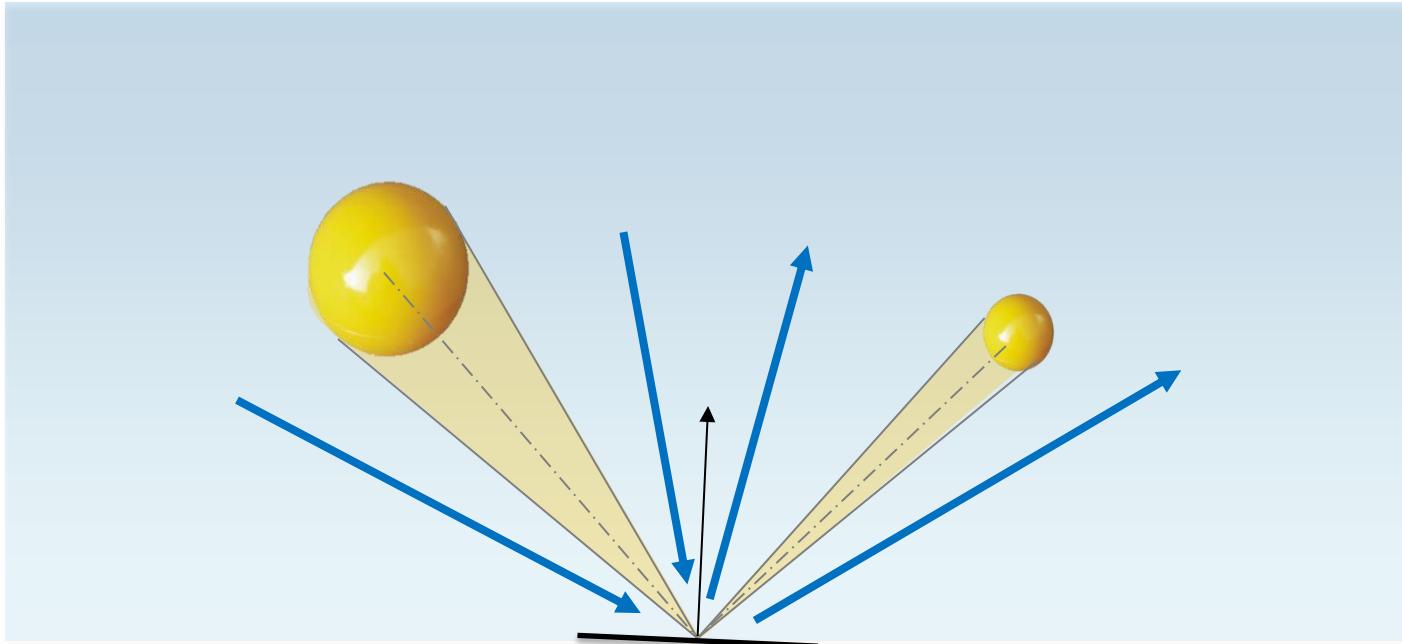
# Using Singular Value Decomposition to Obtain Multi-dimensional Gaussian Flux Distributions for Optical Modelling of Heliostats Images

W.A. Landman, P. Gauché & F. Dinter

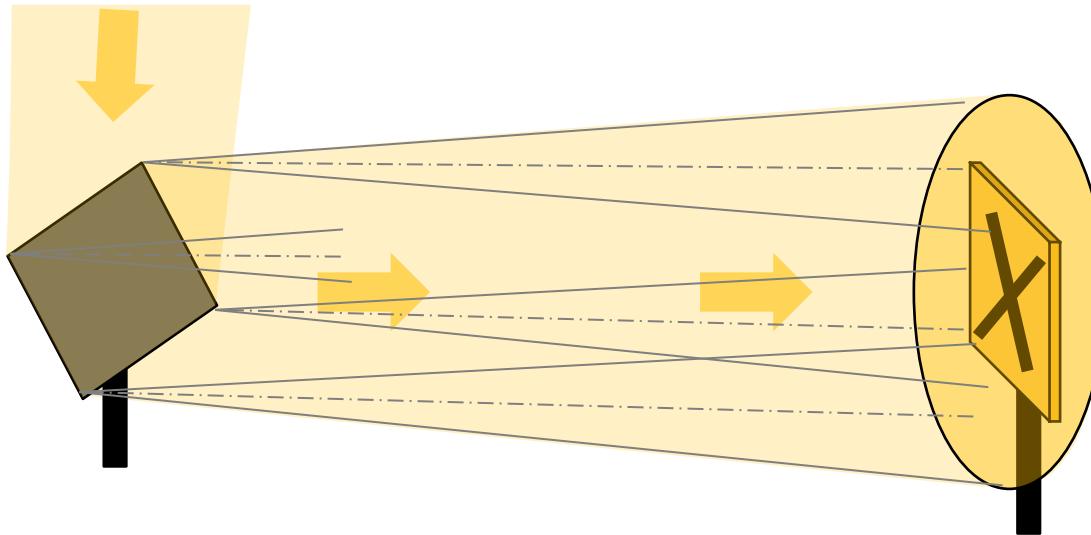
Solar Thermal Energy Research Group (STERG),  
University of Stellenbosch



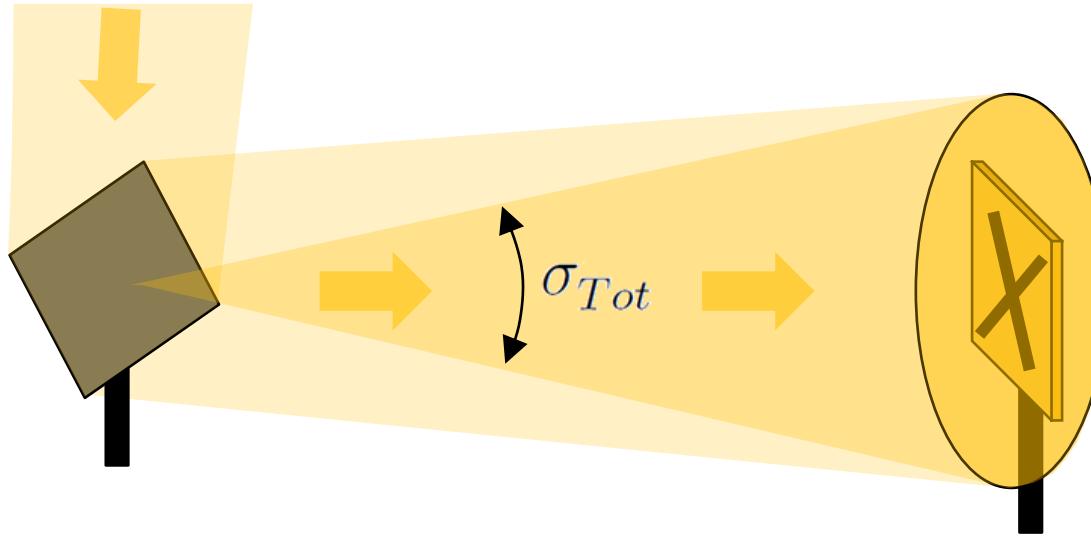
# Snell's Law



# Total Beam Dispersion Error

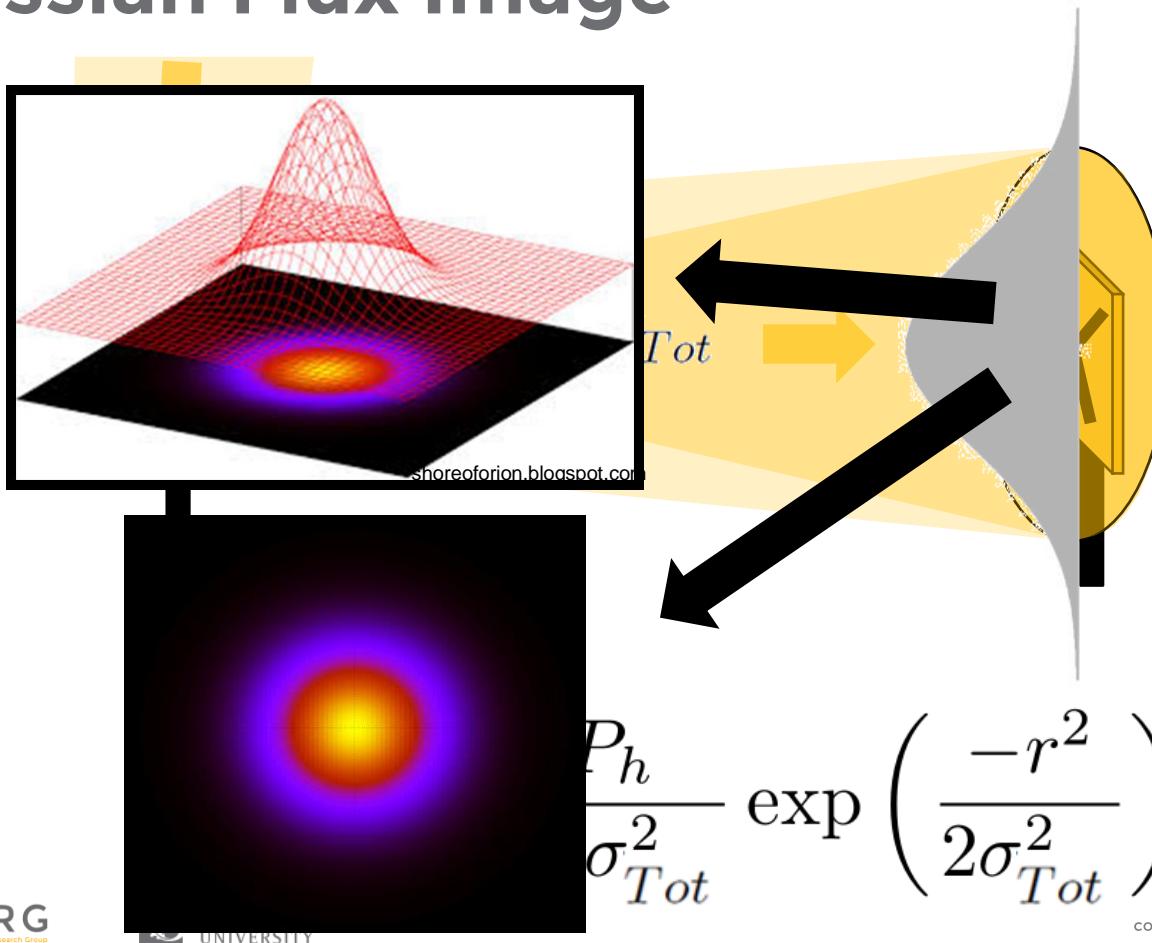


# Total Beam Dispersion Error



$$\sigma_{Tot}^2 = \sigma_{sun}^2 + \sigma_{astigmatism}^2 + \sigma_{BQ}^2$$

# Gaussian Flux Image

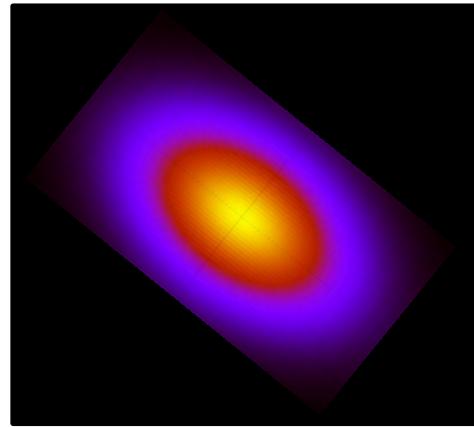
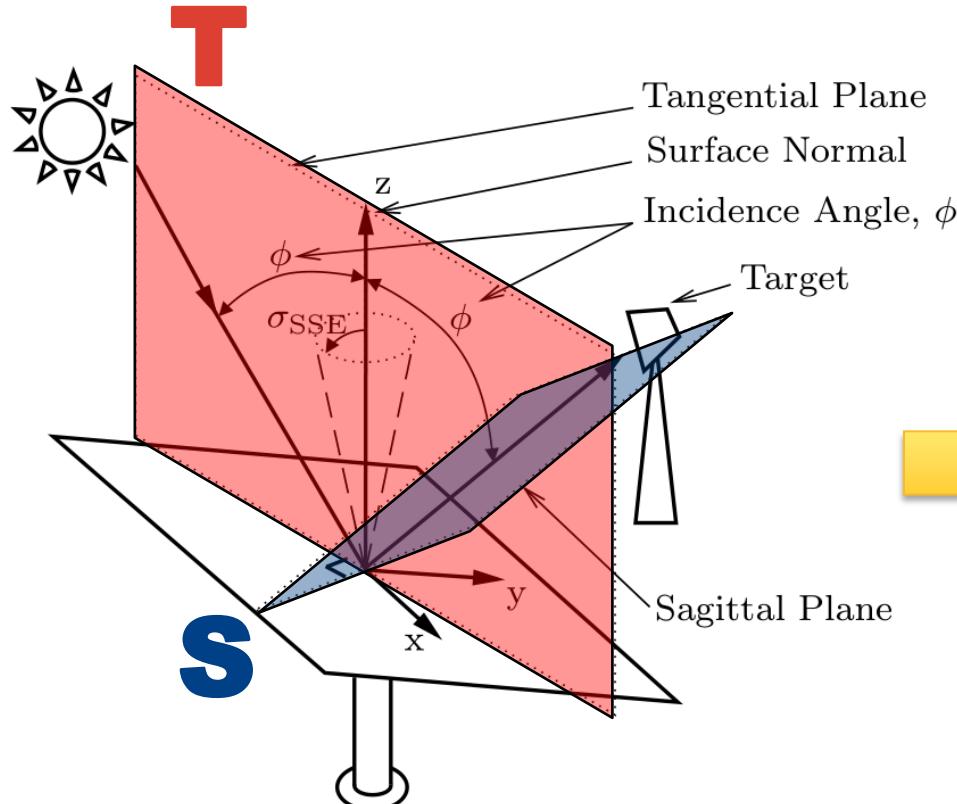


# Total Beam Dispersion Error

## Components:

- Sun Shape (circular)
- Astigmatism (**not** circular)
- Surface Slope Error (**not** circular)
- Specularity Error (circular)

# Elliptical Image



# Axial Convolution



	Tangential	Sagittal
<b>Sun-shape</b> $\sigma_{sun}^2$	$\sigma_{sun}^2$	$\sigma_{sun}^2$
<b>Astigmatism</b> $\sigma_{ast}^2$	$\frac{\left(D \left \frac{d}{f} - \cos \varphi\right \right)^2}{32d^2}$	$\frac{\left(D \left \frac{d}{f} \cos \varphi - 1\right \right)^2}{32d^2}$
<b>SSE</b> $\sigma_{bq}^2$	$4 \sigma_{SSE}^2$	$(1 + \cos^2 \varphi)^2 \sigma_{SSE}^2$

There are 2 unknowns

# Applications?

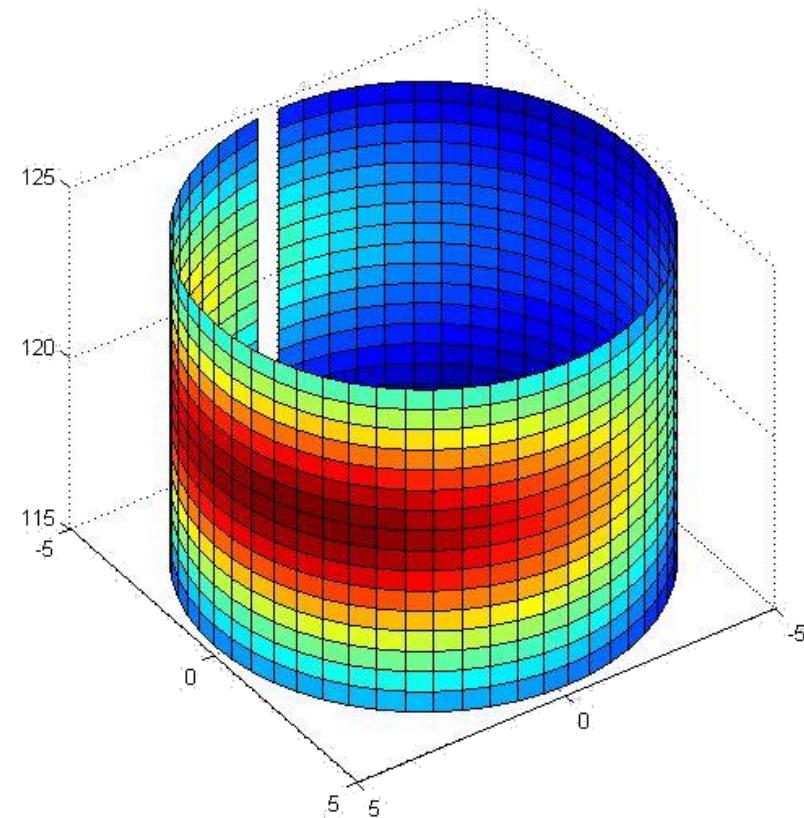
- A) Predict the flux distribution
- OR
- B) From the flux distribution  
determine optical variables

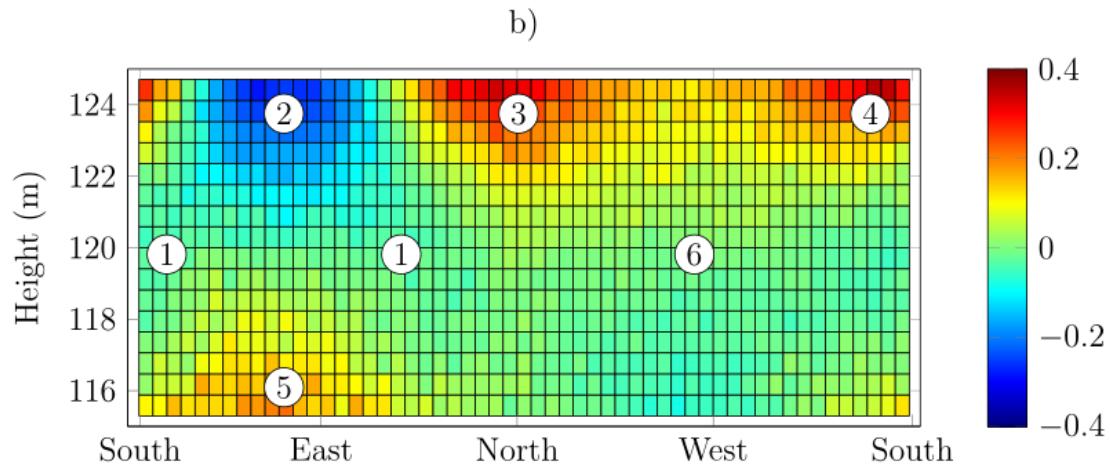
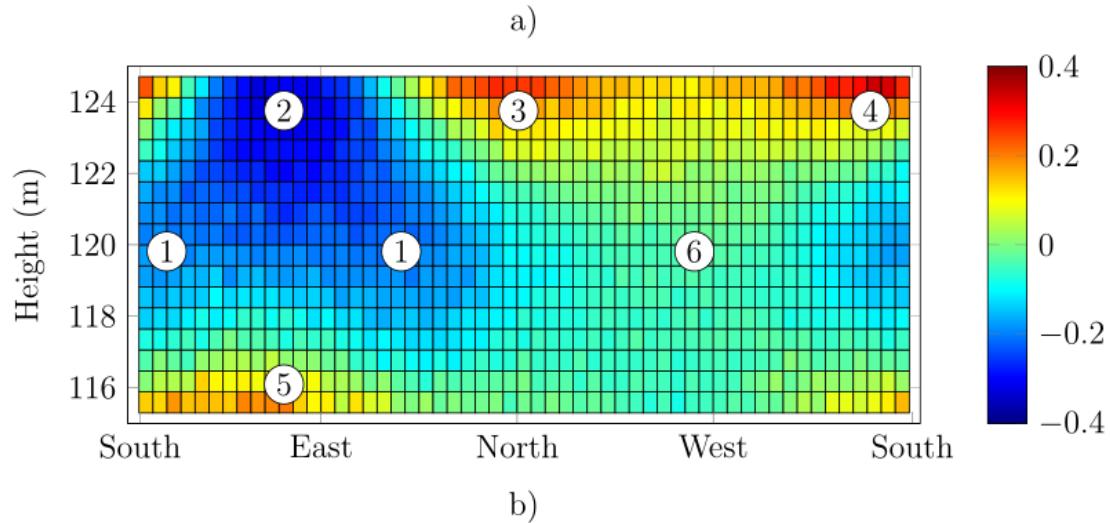
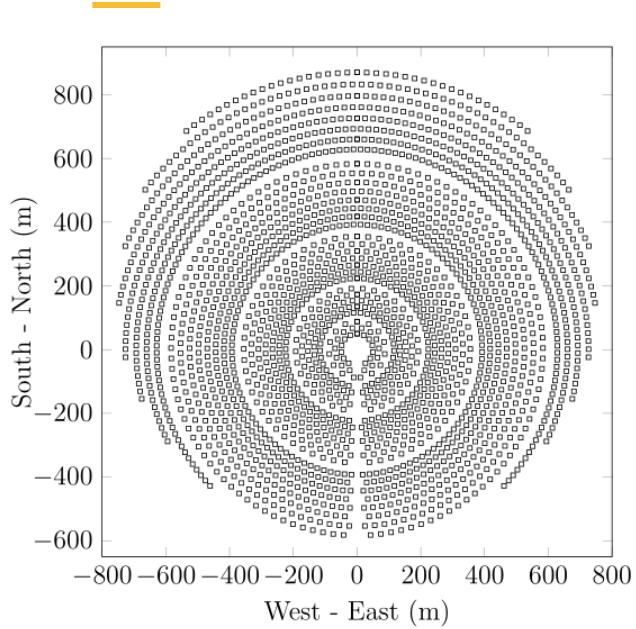
(Montreal, 2013)

# A) Predict the flux distribution

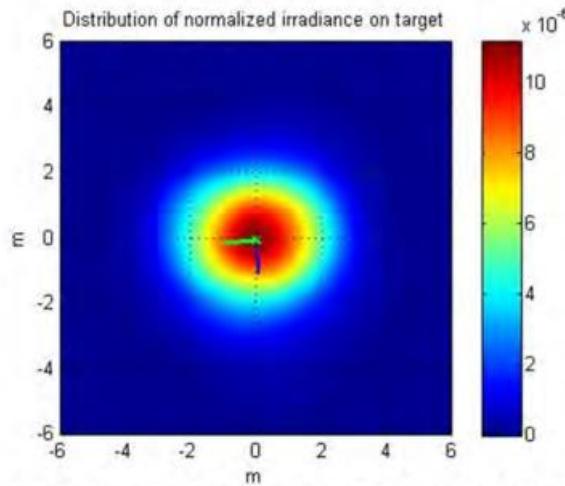


- Speed vs accuracy...
- Now project elliptical images onto complex surfaces at high computational efficiency





# B) Determine optical variables



## STATISTICS ANALYSIS OF REAL IMAGE:

Image centroid = (-0.16, -0.35) m

Image peak = (-0.18, -0.29) m

90-Percent Image Radius = 3.407 m

Horizontal (1 $\sigma$ ) radius  $\mu_x$  = 1.601 m

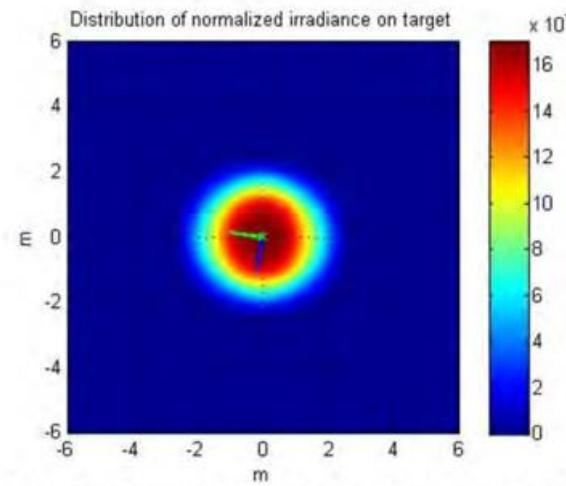
Vertical (1 $\sigma$ ) radius  $\mu_y$  = 1.566 m

Major (1 $\sigma$ ) radius  $\mu_u$  = 1.601 m

Minor (1 $\sigma$ ) radius  $\mu_v$  = 1.566 m

Ellipticity = 1.02

Ellipticity direction = 3.72 deg



## STATISTICS ANALYSIS OF IDEAL IMAGE:

Image centroid = (0.00, 0.00) m

Image peak = (0.00, 0.00) m

90-Percent Image Radius = 2.158 m

Horizontal (1 $\sigma$ ) radius  $\mu_x$  = 1.124 m

Vertical (1 $\sigma$ ) radius  $\mu_y$  = 1.071 m

Major (1 $\sigma$ ) radius  $\mu_u$  = 1.126 m

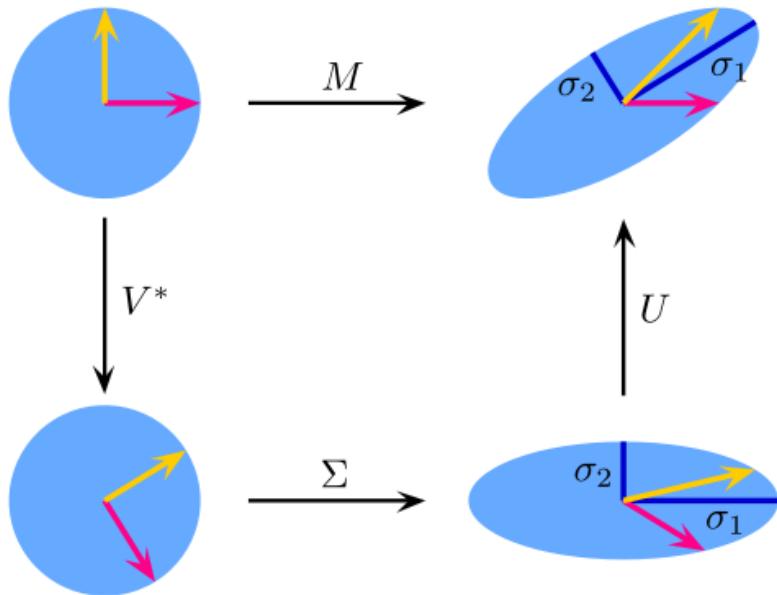
Minor (1 $\sigma$ ) radius  $\mu_v$  = 1.071 m

Ellipticity = 1.05

Ellipticity direction = -9.04 deg

(Montreal, 2013)

# Singular Variable Decomposition

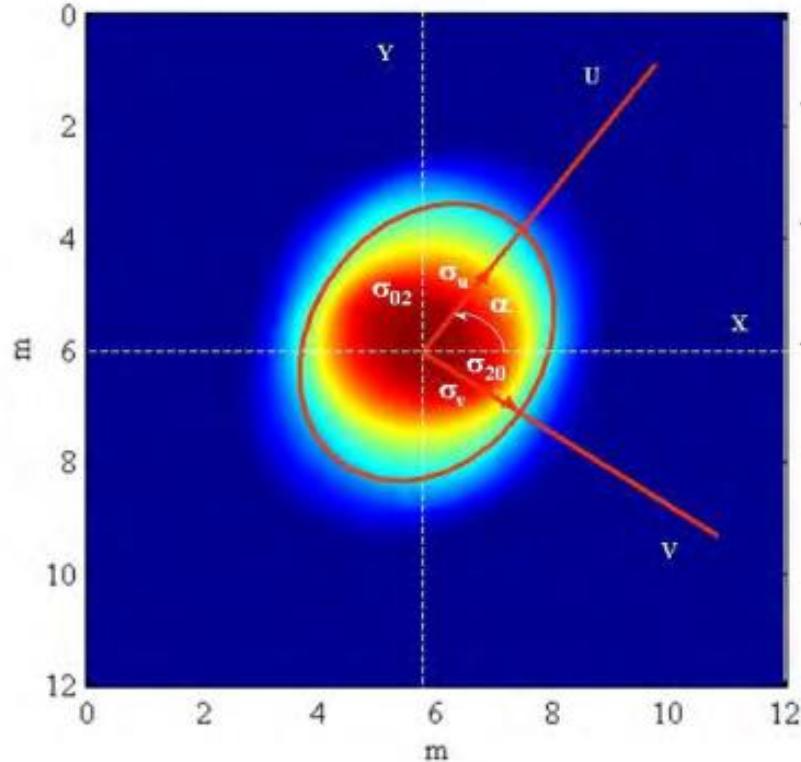


$$M = U \cdot \Sigma \cdot V^*$$

# Singular Variable Decomposition



- Statistical analysis yields:
  - Ellipse major axis
  - Ellipse minor axis
  - Which results in 2 simultaneous equations!



(Montreal, 2013)

# Singular Variable Decomposition

---



- Enables us to link the effects canting and supporting structure on image quality
- Can be automated by incorporating it into the calibration process

# Further Work

---



# Conclusions

---

- SVD is useful for both heliostat image prediction and analysis
- Particular applications are in aim-point optimisation where realtime flux prediction is required

---

## ACKNOWLEDGEMENTS:

### CONTACT DETAILS:

**W Landman**  
Solar Thermal Energy Research  
Group (STERG)  
Stellenbosch University  
South Africa

[STERG@sun.ac.za](mailto:STERG@sun.ac.za)  
+27 (0)21 808 4016

visit us: [concentrating.sun.ac.za](http://concentrating.sun.ac.za)